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METHOD OF SOLUTION OF CERTAIN BOUNDARY VALUE PROBLEMS FOR NONLINEAR HYPERBOLIC EQUATIONS AND PROPAGATION OF WEAK SHOCK WAVES

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A method is given for obtaining exact solutions of certain mixed Cauchy problems for second order nonlinear hyperbolic equations. A detailed study is made using the case of the velocity potential equation corresponding to unsteady, plane-parallel flows of a polytropic gas. This method can be applied to a wider class of equations. Some properties of the solutions obtained are studied. An approximate theory of propagation of curvilinear weak shock waves through a uniform background is constructed to illustrate the application. The investigation commenced in [1] is continued.

1. Let, at the initial instant $t = 0$, a homogeneous polytropic gas in which the speed of sound $c = 1$ be at rest inside or outside a sufficiently smooth, closed, convex cylindrical surface S_0. At the time $t = 0$ a piston S_t begins to move through the gas with zero initial normal velocity V_n and nonzero normal acceleration W_n, compressing or rarefying the gas. No restrictions are imposed on the law of motion of the piston S_t occupying the position S_0 at the time $t = 0$ except the conditions of sufficient smoothness of the law of motion of the convex surfaces of S_t and the conditions already given for V_n and W_n. We seek a solution of a nonlinear equation for the velocity potential $\Phi(x_1, x_2, t)$ [2] in a region bounded by the piston surface S_t and the surface of weak discontinuity R_t which detaches itself at the initial instant from the surface S_0 and propagates with unit normal velocity through the gas at rest.

An approximate solution near the surface R_t for the problem and conditions formulated above was constructed in [1], where the limiting values for the times of existence of smooth potential flows related to the geometry of the surface S_0 and to the law of motion of the piston were found for the gas compression case. We find that the method used in [1] can be employed to construct an exact solution to our problem in the form of a functional series with special independent variables. The question of the region of convergence of this series however, remains open in the general case.

Let us replace the velocity potential $\Phi(x_1, x_2, t)$ by an unknown function $\Psi(u_1, u_2, t)$ according to the formula

$$\Psi = x_1 u_1 + x_2 u_2 - \Phi(x_1, x_2, t) + Mt \tag{1.1}$$

Here M is a constant appearing in the Cauchy equation

$$c^2 = \frac{1}{\varkappa}\left(M - \Phi_t - \frac{1}{2}\sum \Phi_{x_i}^2\right), \qquad \varkappa = \frac{1}{\gamma - 1} \tag{1.2}$$

where c is the speed of sound, u_i are the velocity vector components and γ is the adiabatic index. An equation for Ψ in terms of the variables r, φ, t ($u_1 = r\cos\varphi$, $u_2 = r\sin\varphi$) was obtained in [1].

The formulas for transforming to the physical x_1, x_2, t-space are

$$x_1 = \Psi_r \cos\varphi - r^{-1}\Psi_\varphi \sin\varphi, \qquad x_2 = \Psi_r \sin\varphi + r^{-1}\Psi_\varphi \cos\varphi \tag{1.3}$$

We seek a solution to the mixed Cauchy problem in the form

$$\Psi(r, \varphi, t) = \sum_{k=0}^{\infty} a^{(k)}(\varphi, t)\, r^k \tag{1.4}$$

The plane $r = 0$ corresponding to the motion of the surface R_t of weak discontinuity represents, in the r, φ, t-space, the characteristic manifold for the equation defining the function $\Psi(r, \varphi, t)$. The condition that at $r = 0$ the formulas (1.3) define the motion of the surface R_t moving at the speed of sound equal to unity, yields the following expressions [1]

$$a^{(0)} = \mathrm{const} + \varkappa t, \qquad a^{(1)} = t + f(\varphi) \tag{1.5}$$

where $f(\varphi)$ is an arbitrary function used to define, at $t = 0$, the form of the weak discontinuity R_0 coinciding with the surface S_0

$$x_1 = f(\varphi)\cos\varphi - f'(\varphi)\sin\varphi, \quad x_2 = f(\varphi)\sin\varphi + f'(\varphi)\cos\varphi \tag{1.6}$$

Inserting the series (1.4) into the equation for $\Psi(r, \varphi, t)$ and equating the coefficients of $r^{(s)}$ to zero, we obtain the following equation for $a^{(s+2)}(\varphi, t)$ $(s \geqslant 0)$:

$$(s+1)(s+2)a^{(s+2)} - 2(s+2)(t + f + f'')\, a_t^{(s+2)} = F^{(s+2)}(\varphi, t) \tag{1.7}$$

where

$$\begin{aligned}
F^{(s+2)}(\varphi, t) = {} & \sum_{i,j=0}^{s-2} \{(2i + ij + 1)\, a_\varphi^{(i+2)} a_\varphi^{(j+2)} a_{tt}^{(s-i-j)} - \\
& - (s-1-i-j)(s-i-j)\, a_{\varphi t}^{(i+2)} a_{\varphi t}^{(j+2)} a^{(s-i-j)}\} - \\
& - \sum_{i,j=0}^{s-1} \{(j+1)(j+2)\, a_{\varphi\varphi}^{(i+1)} a^{(j+2)} a_{tt}^{(s+1-i-j)} + \\
& + (i+1)(j+1)(j+2)\, a^{(i+1)} a^{(j+2)} a_{tt}^{(s+1-i-j)} + 2j(i+1)\, a_t^{(i+1)} a_\varphi^{(j+1)} a_{\varphi t}^{(s+2-i-j)}\} + \\
& + \sum_{i,j=0}^{s}{}' \{(i+1)(j+1)\, a_t^{(i+1)} a_t^{(j+1)} a_{\varphi\varphi}^{(s+2-i-j)} + \\
& + (i+1)(j+1)(s+2-i-j)\, a_t^{(i+1)} a_t^{(j+1)} a^{(s+2-i-j)}\} + \\
& + \sum_{i=0}^{s-1} \left\{2(i+1)\, a_\varphi^{(i+1)} a_{\varphi t}^{(s+1-i)} - 2a_\varphi^{(s-1)} a_{\varphi t}^{(i+2)} - \frac{1}{\varkappa}(i+1)(i+2)\, a^{(i+2)} a_t^{(s-i)}\right\} - \\
& - \sum_{i=0}^{s} \left\{\frac{i+1}{\varkappa} a^{(i+1)} a_t^{(s+1-i)} + \frac{1}{\varkappa} a_{\varphi\varphi}^{(i+1)} a_t^{(s+1-i)} + 2(i+1)\, a_t^{(i+1)} a_{\varphi\varphi}^{(s+1-i)} + \right. \\
& \left. + 2(i+1)(s-i+1)\, a_t^{(i+1)} a^{(s-i+1)}\right\} + \frac{1}{2\varkappa}(s-1)s a^{(s)} - \\
& - 2s a_t^{(1)} a_\varphi^{(s+1)} a_{\varphi t}^{(2)} + \frac{s}{2\varkappa} a^{(s)} + \frac{1}{2\varkappa} a_{\varphi\varphi}^{(s)} + a_{\varphi\varphi}^{(s)} + s a^{(s)}
\end{aligned} \tag{1.8}$$

In (1.8) we have $a^{(-k)} \equiv 0$ when $k > 0$, and the prime accompanying the summation sign means that the combination $(i, j) = (0, 0)$ is excluded. Thus the only functions to appear in the right-hand sides of (1.7) are $a^{(k)}$ and their first and second derivatives in φ and t for $k \leqslant s+1$. The general solution of (1.7) has the form

$$a^{(s+2)} = (t+f+f'')^{s+1/2} \left[C^{(s+2)}(\varphi) + \frac{1}{2}\int_0^t F^{(s+2)}(\varphi, t)(t+f+f'')^{-s+3/2} dt\right] \qquad (1.9)$$

where $C^{(s+2)}(\varphi)$ are arbitrary functions. Arbitrary functions appear in the coefficients of $a^{(s)}$ by virtue of the fact that at $r=0$ the Cauchy data give a non-unique solution for the function $\Psi(r, \varphi, t)$, since the plane is the characteristic plane. The functions $C^{(s+2)}(\varphi)$ must be obtained from the prescribed law of motion of the piston surface S_t.

2. We consider the problem of determining the functions $C^{(s+2)}(\varphi)$. Let the motion of the closed surface S_t be defined by the equations

$$x_1 = x_1(\beta, t), \qquad x_2 = x_2(\beta, t) \qquad (2.1)$$

Here β is a parameter such that when $t=0$, $\beta=\varphi$ and the equations (2.1) assume the form (1.9). If the surface S_t represents an impermeable moving wall, then the following kinematic condition must hold on this wall:

$$(\mathbf{u}\cdot\mathbf{n}) = V_n \qquad (2.2)$$

where $\mathbf{u}$ is the velocity vector, $\mathbf{n} = (n_1(\beta, t), n_2(\beta, t))$ is a unit vector normal to S_t and $V_n = V_n(\beta, t)$ is the normal velocity of motion of S_t. Let us expand (2.2). We obtain

$$r_*(\beta, t)(\cos\varphi_*(\beta, t)\, n_1(\beta, t) + \sin\varphi_*(\beta, t)\, n_2(\beta, t)) = V_n(\beta, t) \qquad (2.3)$$

Here $r_*(\beta, t)$ and $\varphi_*(\beta, t)$ are certain a priory unknown functions such that $u_1^* = r_*\cos\varphi_*$ and $u_2^* = r_*\sin\varphi_*$ define the components ($\varphi_*(\varphi, 0) = \varphi$, $r_*(\varphi, 0) = 0$) of the velocity vector on the piston. The functions $r_*(\beta, t)$ and $\varphi_*(\beta, t)$ are found from

$$x_1(\beta, t) = \cos\varphi_* \sum_{k=0}^{\infty}(k+1)\, a^{(k+1)}(\varphi_*, t)\, r_*^k - \sin\varphi \sum_{k=0}^{\infty} a_\varphi^{(k+1)}(\varphi_*, t)\, r_*^k$$

$$x_2(\beta, t) = \sin\varphi_* \sum_{k=0}^{\infty}(k+1)\, a^{(k+1)}(\varphi_*, t)\, r_*^k + \cos\varphi_* \sum_{k=0}^{\infty} a_\varphi^{(k+1)}(\varphi_*, t)\, r_*^k \qquad (2.4)$$

which is obtained by inserting r_*, φ_* and Ψ from (1.4) into (1.3), and the left-hand sides are taken from (2.1). The function $V_n(\beta, t)$ is obtained from the formula

$$V_n(\beta, t) = \frac{D(x_1, x_2)}{D(\beta, t)}\left[\left(\frac{\partial x_1}{\partial\beta}\right)^2 + \left(\frac{\partial x_2}{\partial\beta}\right)^2\right]^{-1/2} \qquad (2.5)$$

The relation (2.3) is an identity in β and t. Let us differentiate it with respect to t and set $t=0$, assuming that all functions entering (2.3) and (2.4) are infinitely differentiable. The corresponding partial derivatives of r_* and φ_* are obtained by differentiating Eqs. (2.4). As the result we obtain a sequence of equations which yield, consecutively, all functions $C^{(s+2)}(\varphi)$ for $s \geqslant 0$ under the conditions that the normal acceleration of the piston $W_n(\varphi, 0) \neq 0$ at $t=0$ and that $f + f'' \neq 0$.

The first differentiation of (2.3) and (2.4) yields the following equations for $C^{(2)}(\varphi)$:

$$\frac{\partial x_1}{\partial t}(\varphi, 0)\cos\varphi + \frac{\partial x_2}{\partial t}(\varphi, 0)\sin\varphi = V_n(\varphi, 0) = a_t^{(1)}(\varphi, 0) +$$

$$+ 2a^{(2)}(\varphi, 0)\frac{\partial r_*}{\partial t}(\varphi, 0) = 0$$

$$\frac{\partial r_*}{\partial t}(\varphi, 0) = W_n(\varphi, 0) \tag{2.6}$$

Equations (2.6) yield the functions $C^{(2)}(\varphi)$ uniquely expressed in terms of $W_n(\varphi, 0)$ and $f(\varphi)$. Next, $C^{(3)}(\varphi)$ is found by superimposing the linear combinations of (2.4) differentiated twice with respect to t

$$\frac{\partial^2 x_1}{\partial t^2}\cos\varphi + \frac{\partial^2 x_2}{\partial t^2}\sin\varphi = 4a_t^{(2)} + 6a^{(3)}\left(\frac{\partial r_*}{\partial t}\right)^2 + 2a^{(2)}\frac{\partial^2 r_*}{\partial t^2} - \\ - \left(\frac{\partial \varphi_*}{\partial t}\right)^2 (a^{(1)} + a^{(1)}_{\varphi\varphi}) + 2a_\varphi^{(2)}\frac{\partial r_*}{\partial t}\frac{\partial \varphi_*}{\partial t} \tag{2.7}$$

where

$$\frac{\partial \varphi_*}{\partial t} = (f + f'')^{-1}\left(\frac{\partial \Gamma}{\partial t} - 2a_\varphi^{(2)}\frac{\partial r_*}{\partial t}\right), \quad \frac{\partial^2 r_*}{\partial t^2} = \frac{\partial^2 V_n}{\partial t^2}(\varphi, 0) \tag{2.8}$$

$$\Gamma = -x_1\sin\varphi + x_2\cos\varphi, \qquad t = 0 \tag{2.9}$$

This process can obviously be continued. The conditions

$$\partial r_*/\partial t\ (\varphi,\ 0) \neq 0, \qquad f + f'' \neq 0$$

holding for the convex surfaces S_0 guarantee the possibility of obtaining unique expressions for any $C^{(s)}(\varphi)$. The expressions for $C^{(s)}(\varphi)$ at $s \neq 3$ are omitted as they are cumbersome.

3. The series (1.4) the coefficients of which were shown above to be uniquely determinable from the prescribed law of motion of the piston S_t, gives an exact formal solution for the problem posed. Determination of the region of convergence of this series and of the series for the corresponding derivatives entering (1.3), present an extremely difficult task.

For the linear hyperbolic systems [3] gives a method for solving the Cauchy problem using convergent expansions in terms of the running waves. In these series generalized functions appear as multipliers and they contain singularities of a lower order than that of the term accompanied. The coefficients of these generalized functions are obtained from ordinary differential equations. The proof of convergence of such series can be reduced to the Cauchy-Kowalewska [4] existence theorem. However it is not at all apparent how these results could be extended to the case of nonlinear hyperbolic equations.

Assuming that the function Ψ has continuous partial derivatives of order $m + 2$ containing the differentials with respect to each of the variables t, r and φ of order not higher than m and neglecting the ($m + 1$)-th term in (1.4), we obtain an approximate solution to the equation for $\Psi(r,\ \varphi,\ t)$. The functions (2.1) are assumed to possess the derivatives of order up to m. The assumptions made hold for a number of real flows [5], for various values of m.

Let us establish the region of convergence of a series analogous to (1.4), for the simplest model case in which the solution of the mixed Cauchy problem is obtained using the method proposed for the one-dimensional wave equation

$$u_{tt} - u_{xx} = 0 \tag{3.1}$$

Let the initial and the boundary conditions have the form

$$u(x, 0) = u_t(x, 0) = 0, \qquad x \geqslant 0, \qquad u_x(0, t) = F(t), \qquad t \geqslant 0 \tag{3.2}$$

Introducing a new unknown function $\Psi(r,\ t)$ according to the formula $\Psi = xu_x - u$

$(r = u_x)$. we obtain the Monge-Ampère equation

$$\Psi_{rt}^2 - \Psi_{rr}\Psi_{tt} = 1, \qquad x = \Psi_r \tag{3.3}$$

Seeking the solution to our problem in the form

$$\Psi(r, t) = \sum_{k=0}^{\infty} A^{(k)}(t)\, r^k \tag{3.4}$$

we find that $A^{(0)}(t) = 0$ and $A^{(1)}(t) = t$, since the line $r = 0$ corresponds to a weak discontinuity propagating with a unit velocity. For $s \geqslant 2$ the coefficients $A^{(s)}(t)$ are found from the following equations $(k \geqslant 1)$:

$$\sum_{i=0}^{k} (i+1)\,[(k-i+1)\, A_t^{(i+1)} A_t^{(k-i+1)} - (i+2)\, A^{(i+2)} A_{tt}^{(k-i)}] = 0 \tag{3.5}$$

From (3.5) we find by induction that all $A^{(s)}(t) = C_s$ $(s \geqslant 2)$, where C_s are arbitrary constants. Consequently the series (3.4) has the form

$$\Psi(r, t) = tr + \sum_{s=2}^{\infty} C_s r^s \tag{3.6}$$

The constants C_s can be found from the following equation derived from (3.2)

$$t + \sum_{s=2}^{\infty} sC_s\, [F(t)]^{s-1} = 0 \tag{3.7}$$

Let us set $F(t) = \omega$ and assume $F(t)$ to be such, that a unique inverse function F^{-1} $(F^{-1}(0) = 0)$ exists near the point $t = 0$

$$t = F^{-1}(\omega) = \sum_{k=1}^{\infty} \alpha_k \omega^k \tag{3.8}$$

and the series (3.8) converges absolutely in the region $|\omega| < \omega_0$. Then (3.7) and (3.8) imply that $C_{s+1} = -\alpha_s / s+1$, and the series (3.6) as well as the series for the derivatives all converge whenever the series for $F^{-1}(\omega)$ converges.

In addition to establishing the convergence of the series in question, we must also establish the feasibility of the passage to the physical x_1, x_2, t -space, i.e. the solvability of the Eqs. (1.3) with respect to the functions $r = r(x_1, x_2, t)$ and $\varphi = \varphi(x_1, x_2, t)$. To do this, it is sufficient to verify that the Jacobian $I = \partial(x_1, x_2) / \partial(r, \varphi)$ does not vanish within the time interval $0 < t < T$. Having computed I for a fixed value of t, we obtain

$$I = r^{-1}(\Psi_{r\varphi} - r^{-1}\Psi_\varphi)^2 - \Psi_{rr}(\Psi_r + r^{-1}\Psi_{\varphi\varphi}) \tag{3.9}$$

The form of I given by (3.9) implies at once that if the flow is not too dissimilar from a one-dimensional flow ($\Psi_{r\varphi}$, Ψ_φ and $\Psi_{\varphi\varphi}$ are small), then a passage to the physical x_1, x_2, t -space near the surface R_t is possible as $\Psi_{rr}(0, t)$ and $\Psi_r(0, t) \neq 0$.

4. Let us make use of the expressions for the functions derived in Sect. 2 to investigate the propagation of weak shock waves arising, after the disruption of the potential flows, directly at the surface R_t of the weak discontinuity, for the case of compressive flows (the piston S_t moving into the gas).

Formulas (1.3) imply that the gradients of the gasdynamic quantities become infinite at the surface R_t when $\Psi_{rr}(0,\varphi,t)$ vanishes. Thus, having found the curve $t = t(\varphi)$ from the equation $a^{(2)}(\varphi, t) = 0$ we obtain, in the x_1, x_2, t-space, a three-dimensional curve Γ on the surface R_t at which a shock wave begins to form with zero initial intensity.

To find the law of motion of the shock wave across a constant background, we shall use the following segment of the series (1.4)

$$\Delta = a^{(0)} + a^{(1)}r + a^{(2)}r^2 + a^{(3)}r^3 \tag{4.1}$$

in which $a^{(i)}(\varphi, t)$ is defined by (1.9). We assume that the shock wave is weak and that the flow behind it is isentropic, potential, and described by (4.1). As we have already noted in [1], the term $a^{(3)}r^3$ must be retained for the transfer of the profiles of the quantities behind the shock wave.

The motion of the shock wave front in the x_1, x_2-plane will be described by the formulas (1.3) in which Ψ is replaced by Δ from (4.1) and set $r = R(\varphi, t)$, where R is a function to be defined. The equation for $R(\varphi, t)$ can be obtained by finding the normal velocity of motion $D^*(\varphi, t)$ of the shock wave described by the formulas $x_1 = x_1(\varphi, t)$, $x_2 = x_2(\varphi, t)$ which follow from (1.3), and equating this velocity to the expression

$$D(\varphi, t) = 1 + \frac{\gamma+1}{4}R + \frac{(\gamma+1)^2}{32}R^2 \tag{4.2}$$

where $D(\varphi, t)$ is the normal velocity of motion of the shock wave obtained from the Hugoniot conditions [6].

Performing the computations we obtain, from the relation $D^{*2} = D^2$ the following first order nonlinear partial differential equation for the function $R(s,t)$:

$$D^2[(\Delta_{rr}R_\varphi + AR)^2 + (B + AR_\varphi)^2] =$$
$$= \left[\left(\Delta_{rt}A - \Delta_{rr}\frac{\Delta_{\varphi t}}{R}\right)R_\varphi + (\Delta_{rr}B - RA^2)R_t + (\Delta_{rt}B - \Delta_{\varphi t}A)\right]^2 \tag{4.3}$$
$$\Delta_{r\varphi} - \frac{\Delta_\varphi}{R} = RA, \qquad \Delta_r + \frac{\Delta_{\varphi\varphi}}{R} = B$$

The initial condition $R = 0$ holds for (4.3) along the line l [1]

$$t = \frac{1}{(\gamma+1)^2 W(\varphi)}\left[\frac{1}{W(\varphi)(f+f'')} - 2(\gamma+1)\right] \tag{4.4}$$

where $-W(\varphi) > 0$ is the normal acceleration of the piston S_t at $t = 0$, and it is assumed that no focusing takes place at the surface R_t (the radii of curvature of R_t do not vanish).

The independent term in the left-hand part of (4.3) (not containing R) is equal to B^2 and also to the independent term in the right-hand side of (4.3), as $\Delta_{rt} = 1 + O(r)$. Therefore (4.3) has a solution $R \equiv 0$ satisfying the initial condition $R = 0$ along the curve l.

Having computed the possible values of the normal derivative of R along the curve l and reducing the solution of the problem, in the usual manner, to a solution of a system of ordinary differential equations, we can show that when certain restrictions are imposed on the classes of flows, only one nonzero solution of (4.3) exists, apart from the zero solution near l, satisfying all the necessary conditions and having a physical meaning.

5. Let us consider in more detail the one-dimensional propagation of weak cylindrical shock waves through a stationary gas.

Let a cylindrical piston, which at the initial instant has a zero velocity and nonzero acceleration $-W > 0$ begins to enter a homogeneous gas flowing at that instant with the speed of sound $c = 1$ outside a cylindrical surface of radius R_0. The instant t^* of destruction of the potential flow at the surface of weak discontinuity is determined by formula (4.4).

Introducing the variable $x = t + R_0$ we obtain from (4.3), after computing $a^{(2)}(\varphi, t)$ and $a^{(3)}(\varphi, t)$ the following ordinary differential equation for $R = R(x)$:

$$R' \{C_2 x^{1/2} + (\gamma + 1) x + R\, [Ax - {}^1/_2 (\gamma + 1)(\gamma + 4) x \ln x + {}^{11}/_8 C_2^2 + {}^1/_4 C_2 (15\gamma + 27) x^{1/2}]\} + R\, [{}^1/_2 C_2 x^{-1/2} + {}^3/_4 (\gamma + 1)] + {}^1/_2 R^2 [A - {}^1/_2 (\gamma + 1)(\gamma + 4)(\ln x + 1) + {}^1/_8 C_2 (15\gamma + 27) x^{-1/2} - (\gamma + 1)^2/16] = 0 \qquad (5.1)$$

where the constant C_2 can be found from (2.6) and is

$$C_2 = R_0^{-1/2} W^{-1} (1 - (\gamma + 1) R_0 W) < 0 \qquad (\partial r_* / \partial t = -W) \qquad (5.2)$$

and the constant $A = {}^1/_6 C_3$ is defined from (2.7) using the prescribed value for the derivative of the acceleration at $t = 0$.

The initial point $(R, x) = (0, x^*) = (0, t^* + R_0)$ for (5.1) is a saddle point and two integral curves pass through it. One of them is $R \equiv 0$ and the other curve has a positive slope at the point $(0, x^*)$ provided that the constants C_2, A and x^* satisfy the inequality

$$Ax^* - {}^1/_2 (\gamma + 1)(\gamma + 4) x^* \ln x^* + {}^{11}/_8 C_2^2 + {}^1/_4 C_2 (15\gamma + 27) x^{*1/2} < 0 \qquad (5.3)$$

When the values of the constants W, R_0, A and γ are fixed, a nonzero solution of (5.1) passing through the point $(0, x^*)$ can be constructed by numerical method. When conditions (5.3) hold, the integral curves have a single maximum on the interval $[x^*, \infty)$ and decrease to zero at $x \to \infty$.

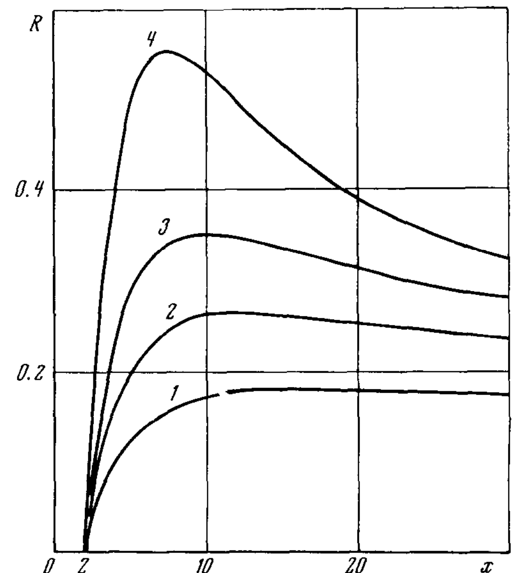

Fig. 1

Figure 1 gives the numerical results computed for $\gamma = 1.4$, $W = -1$, $R_0 = 1$ and for the values of A equal to 15, 19, 21 and 23, the latter corresponding to the curves 1—4 respectively. The maximum values of R and the corresponding values of x are: (0.182, 18.3), (0.263, 12.8), (0.344, 10.1) and (0.547, 7.3). At the critical value $A = A^* \approx 25.4$ the inequality (5.3) violates.

In conclusion, let us investigate the problem of the asymptotic law of decay of weak cylindrical shock waves over long periods of time (at long distances from the wave origin).

Assuming that $R \to 0$ as $x \to \infty$ and estimating the order of the terms in

(5.1) in the first approximation we obtain, neglecting the higher order terms, the following equation:

$$R'x + {}^3/_4 R = 0 \tag{5.4}$$

which yields asymptotic law of decay of weak shock waves $R = O\,(x^{-3/4})$ established by Landau [7]. Expressing further $R\,(x)$ in the form

$$R\,(x) = Cx^{-3/4} + Q\,(x), \qquad C = \text{const} \tag{5.5}$$

and estimating the order of the terms in the equation for $Q\,(x)$ obtained from (5.1) we arrive, in the second approximation, at the following equation (assuming that $\lim Q\,(x)\,x^{3/4} = 0$ when $x \to \infty$):

$$(\gamma + 1)\,xQ' + {}^3/_4\,(\gamma + 1)\,Q - {}^1/_4\,CC_2 x^{-5/4} = 0 \tag{5.6}$$

The latter can be used to obtain the following law of decay of the weak cylindrical shock waves at large t

$$R \sim C\left[(t + R_0)^{-3/4} - \frac{C_2}{2\,(\gamma + 1)}\,(t + R_0)^{-5/4}\right]$$

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